Generative Grammar Based on Arithmetic Operations for Realtime Composition

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Abstract. Mathematical sequences in $\mathbb{N}_0$ are regarded as time series. By repeatedly applying arithmetic operations to each of their elements, the sequences are metamorphised and finally transformed into sounds by an interpretation algorithm. The efficiency of this method as a composition method is demonstrated by explicit examples. In principle, this method also offers laypersons the possibility of composing. In this context it will be discussed how well and under what kind of conditions the compositional results can be predicted and thus can be deliberately planned by the user. On the way to assessing this, Edmund Husserl's concept of "fulfillment chains" provides a good starting point. Finally, the computer-based board game MODULO is presented. Based on the here introduced generative grammar, MODULO converts the respective game situation directly into sound events. In MODULO, the players behave consistent to the gaming-rules and do not care about the evolving musical structure. In this respect, MODULO represents an alternative draft to a reasonable and common use of the symbols of the grammar in which the user anticipates the musical result.

Keywords: algorithmic composition, phenomenology, arithmetic operations, realtime composition, live coding, Edmund Husserl, notation system

1 Introduction

This thesis deals with a generative process in the field of real-time composition, which is essentially based on the fact that different arithmetic operations are repeatedly applied to the elements of a mathematical sequence. In the following, this basic procedure shall be abbreviated as **AOG** (**A**rithmetic-**O**peration-**G**rammar).

"Every human is a composer" – with this casual modification of a saying by Joseph Beuys I would like to express that generative composition processes basically open up the possibility that even people with little knowledge of music theory can compose, since in the sense of Chomsky's division of generative grammars those of level 3 – the one presented here is one of this kind – help to produce exclusively meaningful/wellformed musical structures [2], [3].

Typically, generative methods of composition are judged from the point of view of what kind of structures they produce and if so what relation they have to music [17].

However, in the second part of this work, the actual process is discussed under another aspect, namely the extent to which the generative process chain can also be mapped in the mind of a person who produces it, especially with regard to its possible use as a composition aid for laypersons. To even consider taking such a direction is motivated by the fact that the overall procedure presented here works in such a way that the process of generating the composition from its symbolic representation is straightforward, without the need for automatic corrections or optimizations of the linear or harmonic structure. At least this ensures a relative transparency of the generating process.

But first the actual procedure is described in detail both theoretically and in examples and its special characteristics are analyzed:

2 A 3rd order generative grammar based on arithmetic operations applied to mathematical sequences

The overall shape of a sequence such as $a_{i+1} = a_i + 1$ (identity on natural numbers), or $a_{i+1} = a_i + a_{i-1}$ (fibonacci sequence) is to be changed by applying an arithmetic operation to each of its sequence members. This can be repeated on the resulting sequence with another operation, and so on. One gets a metamorphosis of sequences which have a close structural relation to each other.

For musical use, from now on all sequences are to be understood as time sequences which deliver their values in a fixed time interval ΔT within a real-time composition process.

Restrictively, initially only $id(\mathbb{N}_0) = \{0, 1, 2, 3, 4, ...\}$ is to be used as a source or time base, to which the arithmetic operations are subsequently applied.

$\mathbb{N}_0$ is also the permitted number range. So that this number range is never left, a filter is set after the execution of any operation, in which the decimal places are truncated and values smaller than zero are set to zero. In table 1 some operators are suggested to be used for this grammar. There the symbols used for the operations and their meaning are shown together with an example. In addition, it is shown here how the corresponding operator is represented in the game "MODULO" introduced at the end of this presentation.

The operators proposed here go a little beyond of what is common in arithmetic. In order to understand the table, the operators $\neq$, $==$, $\nmid$, $\mid$ should also be regarded as a type of filter that allows a number to pass when the condition meant is fulfilled.

symbol	symbol in MODULO	meaning	example
$+$	$+$	addition	$\{0,1,2,3,4\}+3=\{3,4,5,6,7\}$
$-$	$-$	subtraction	$\{0,1,2,3,4\}-3=\{0,0,0,0,1\}$
$\cdot$	$\cdot$	multiplication	$\{0,1,2,3,4\}\cdot 2=\{0,2,4,6,8\}$
$\neq$	$++$	not equal	$\{0,1,2,3,4\}\neq 3=\{0,1,2,0,4\}$
$==$	$--$	identity	$\{0,1,2,3,4\}==3=\{0,0,0,3,0\}$
$\div$	$\cdot\cdot$	division	$\{0,1,2,3,4\}\div 2=\{0,0,1,1,2\}$
$\nmid$	$+++$	does not divide	$\{0,1,2,3,4\}\nmid 2=\{0,1,0,3,0\}$
$\equiv$	$---$	modulo	$\{0,1,2,3,4\}\equiv 3=\{0,1,2,0,1\}$
$\|$	$\cdots$	true divider	$\{0,1,2,3,4\}\|2=\{0,0,2,0,4\}$

Table 1: Used operators with examples.

2.1 Sound generation on the basis of a mathematical sequence

For sound generation, $id(\mathbb{N}_0)$ is now executed as a counting process with constant speed. The introduced grammar makes it easy to gradually increase the complexity of simple structures by adding an operation. Thus, adding a symbol on the symbol level typically results in an increase of complexity at the score level.

Each intermediate result of the successive operations is used in parallel for the sound generation. Thus, the members of each resulting sequence, including those resulting from the intermediate operations, are regarded as divisors of the base number b, with for example $b = 2520 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$.

Whenever the divisibility is actually given and a number between for example 55 and 1760 (A1 and A6) comes out, this is understood as frequency, which is then mapped in the best possible way to the equally tempered scale, so that this tone can then be played in real time e.g. as a piano tone by a sequencer. This mechanism plays the role of a filter that suppresses pitches that have a too large harmonic difference to the overall structure.

2.2 "$\equiv 7 \cdot 5 \equiv 3 \cdot 5$" – a simple composition as an example

"$\equiv 7 \cdot 5 \equiv 3 \cdot 5$" is meant as a symbolic representation of a tiny composition (for sound and complete score see [7]). As it is a convention to apply all operators to $id(\mathbb{N}_0)$ first this information can be neglected in the symbolic representation. As an additional convention one operation is applied after the other with a time delay of twelve times ΔT which can be interpreted as two three-four time bars. Figure 1 shows how the unfolding of this composition could take place starting from the symbolic representation. Obviously, as we go through the successive stages of the unfolding process, there is a steady increase of information and complexity in the resulting structure.

2.3 Analysis of the Musical Structure

At first glance the resulting musical structure seems to be very similar to (repetitive) minimal music. This will be analysed in more detail here.

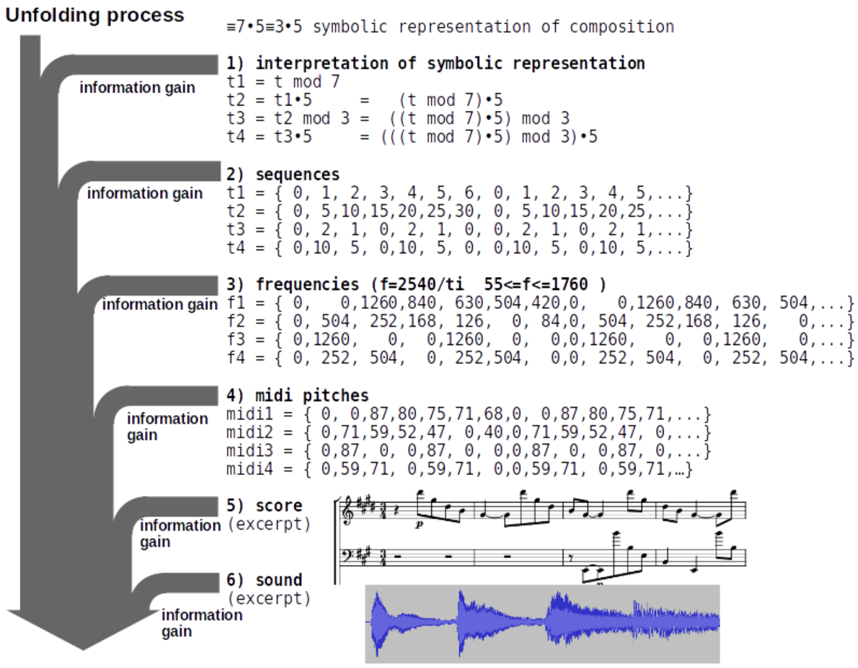

Fig. 1. Unfolding process from symbolic representation to sound.

First of all, the musical structure does not have to be analyzed at the level of the score, but it already becomes apparent after all mathematical operations have been applied, but before the resulting sound events are determined. These do not yet represent sound events, but indices of potential sound events (see Figure 1). As a result of the successively applied operations, generally several superimposed structures appear. By looking at the individual intermediate results, one already obtains an analytical view of the structure without additional effort.

Some operations can easily be related to known musical forms, for example a subtraction applied to $id(\mathbb{N}_0)$ corresponds to the emergence of the same sequence only time-delayed and thus to the structure of an imitation canon, e.g. $\{0, 1, 2, 3, 4, 5, 6, 7, 8, ...\} - 2$ results in $\{0, 0, 0, 1, 2, 3, 4, 5, 6, ...\}$.

As already mentioned above, values smaller than zero are always set to zero and decimal places are neglected.

The modulo division is mainly responsible for the repetitive structures that frequently occurs here, e.g. $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ...\}$ *modulo* 5 results in

$\{0, 1, 2, 3, 4, 0, 1, 2, 3, 4, ...\}$.

The division applied to $id(\mathbb{N}_0)$ results in a slowed down sequence of the same indices when successive identical indices are joined together, e.g.

$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ...\}/2$ results in $\{0, 0, 1, 1, 2, 2, 3, 3, 4, 4, ...\}$. Thus, the structures occurring during division show a certain similarity with the musical structure with the musical form of an augmentation canon.

On the whole, the actual minimal music effect results from the fact that the surgery through a newly applied operation always torments the entire picture to a not too extreme extent, instead of changing individual things in isolation.

2.4 Analysis of the Harmonic Structure

The creation of musical structures with **AOG** is constructive. There is no harmonic analysis and no correction of the harmonic interactions. This is also not necessary for **two** reasons:

The individual numbers in the sequence obtained from an arithmetic operation are used as divisors of the so-called base number. Thus, these numbers result in a certain picking of prime factors from the base number. The product of the selected prime factors – respectively the result of the division – is then interpreted as the frequency of the tone to be heard. Finally, this frequency is mapped to the tempered tone system.

The frequencies that can be generated in this way have only a limited degree of dissonance to each other. Leonhard Euler has already provided a method to measure this. He called his method "gradus suavitatis" g. It is very well suited to this task because, like the approach here, it is based on integer frequencies f, which are then broken down into their prime factors p_i to obtain their degree of dissonance g: For $f = \prod_{i=1}^{n} p_i^{k_i}$ the "gradus suavitatis" is $g = 1 + \sum_{i=1}^{n} p_i.k_i - \sum_{i=1}^{n} k_i$ [1]. (The much discussed problems in the application of the gradus to classical harmony theory will be ignored in this context.)

The degree of dissonance between two frequencies is then the gradus function for the prime factors in which the two compared frequencies differ from each other. Since the prime factors of the base number consist of relatively small prime numbers, it is immediately obvious that when comparing two frequencies that can be generated from, only relatively small degrees of dissonance are produced according to the gradus function.

In addition to this fundamental limitation of the degree of dissonance, a **second factor** that plays a role, that the harmonic event that results in **AOG** generally seems to be reasonable. It can be found in a meaningful organization not only of sound events, but also of their harmonical relationships by the algorithm.

It is anything but trivial to explain here what makes sense and what does not. Since the examples of Bach's monophonic polyphony and at the latest since the tintinabuli harmony of Arvo Pärt, it is clear that even sound events that are far apart can be related melodically or harmonically to each other if they are in the same register in the first case and even not in the second.

Since the sequences of numbers resulting from the arithmetic operations are applied as divisors of the basic number used, **AOG** does not only result in a multi-level rhythmic musical structure right from the start, but they also bring the harmonic relationships of the tones into a rhythmic order.

This principle will be illustrated in a small (a bit academic) example: The base number is $b = 2{\cdot}3{\cdot}5 = 30$. For the sequence $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ modulo 7 is applied. The result is the sequence $\{0, 1, 2, 3, 4, 5, 6, 0, 1, 2, 3, 4, 5, 6\}$.

Each sequence element is used as a divider of b. Where this is not possible, a zero remains (no tone). This results in the following frequency sequence: $\{0, 2 \cdot 3 \cdot 5, 3 \cdot 5, 2 \cdot 5, 0, 2 \cdot 3, 5, 0, 2 \cdot 3 \cdot 5, 3 \cdot 5, 2 \cdot 5, 0, 2 \cdot 3, 5\}$.

If one looks at the prime factors of the occurring tone frequencies separately, it turns out that not only the frequencies of the sound events themselves occur in rhythmic order, but also the individual prime factors of these frequencies: $\{0, 2, 0, 2, 0, 2, 0, 0, 2, 0, 2, 0, 2, 0\}$, $\{0, 3, 3, 0, 0, 3, 0, 0, 3, 3, 0, 0, 3, 0\}$,
$\{0, 5, 5, 5, 0, 0, 5, 0, 5, 5, 5, 0, 0, 5\}$.

2.5 Musical Interpretation and Sound Generation

In principle, the fact that the entire intermediate stages of the generative process on the way from the symbolic representation to the representation of the sequence of the sound events are available offers a multitude of starting points for controlling musical parameters in the field of musical interpretation. In particular, it is possible to take into account by which partial sequence of the applied operations a certain tone was produced.

However, since this work is currently less focused on this last step of musical interpretation, a rather minimalist procedure was initially applied here: Each note is assigned a sample of a percussive instrument. If it turns out that the same frequency has to be played several times simultaneously at a certain point in time, these events are just played in combination and thus form acoustically one event with a corresponding increase of volume. The whole software was implemented in Java (Processing) and for the actual sound generation a simple sequencer, which is also implemented in Java, is responsible, which allows to stream wav files (also superimposed).

3 The Concept of Transparency Considering Generative Grammars

> " *I am giving a performance in Toronto ... I call it Reunion. It is not a composition of mine, though it will include a new work of mine, 0'00" II, ...* [10]. "

John Cage represents to an extreme degree an attitude towards the work in which the maker, the composer, steps back behind the work. This attitude can be read from his late works in that arrangements of things found by chance often form the basis for a musical structure. This basic attitude has strongly influenced the art world both in the visual arts and in music, and the trend is that the composer is no longer the creator of a musical structure, but determines the setting in which the composition then happens [13]. Especially in the field of algorithmic composition there is the widespread basic attitude that the composing subject has no direct imaginative access to what the algorithm itself produces. During the discussion on [18] Sever Tipei notes that music is experimental for him when the result of the generation process is unpredictable. One may or may not follow this paradigm, the fact is that the creation of a setting creates a certain void, which is then often filled by interaction with the (active) recipients. And it is also a fact that these people who are involved in the artistic process bring their own ideas about what music or art is. If one admits this and takes it seriously, and thus gives human interaction a higher meaning than that of a mere random generator, the question immediately arises to what extent the setting provided allows the active recipient to consciously design a (musical) performance according to his or her own ideas.

As mentioned above, in terms of Chomsky's division of generative grammars, **AOG** is one of level 3: Its application ensures that only meaningful/well-formed musical structures are created ([2], [3]) and can thus in principle also enable people with little knowledge of music theory to compose. On the other hand, this advantage is bought at the price of a certain lack of transparency with regard to the relationship between a sequence of symbols and the musical form they represent, and is thus directly opposed to the claim of being able to mentally foresee the resulting musical structures.

Can this shortcoming in **AOG** somehow be compensated by the fact that we are already well versed in dealing with arithmetic operations and infinite sequences, which together form the basis for **AOG**, due to our school education in general? So does this kind of mathematical education in **AOG** allow us to mentally understand the connection between symbolic representation and the musical form it represents?

In order to prepare an answer to this question by first gaining an approximate understanding of how a corresponding mental process can be imagined, a suitable description Husserl's will first be referred to below. It deals with the mental process of how we obtain out of an arithmetic term an idea of the set represented by it.

3.1 Husserl's "Philosophy of Arithmetic"

Husserl's "Philosophy of Arithmetic" comes from a time before he founded his phenomenological method.

The starting point for the development of mathematical concepts in this text is the set as a phenomenon directly accessible to man.

This fact alone should legitimize a deliberately phenomenological reading of this early text, as it is carried out here below. This attitude is also supported by the work of Lohmar [12], and also by the fact that Husserl again, in his later work "logical investigations" ("*Logische Untersuchungen*"), which co-founded the phenomenological method, cites the example of the mental unfolding of mathematical expressions down to the set (see below) to illustrate the difference between the instant imagination of a phenomenon ("*eigentliche Vorstellung eines Phänomens*") and a symbolically intermediated imagination ("*uneigentliche Vorstellung eines Phänomens*") [6].

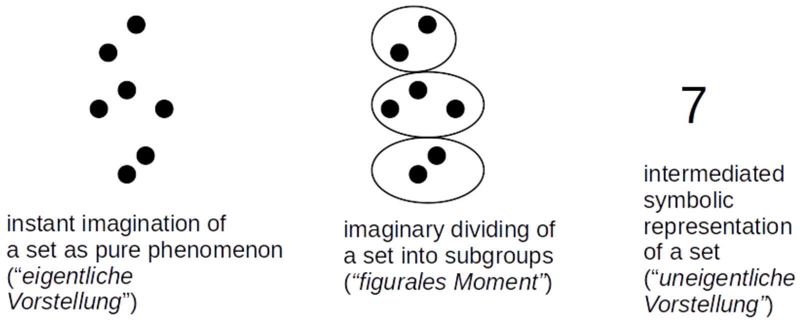

Fig. 2. How to imagine sets.

3.2 Imaginating Sets

Husserl regards sets as an elementary phenomenon. He emphasizes that an instant imagination of sets is possible [5, 201-203], but only for very small sets. And even for very small sets, we still manage by dividing them into subgroups in order to capture their extent ("*figurales Moment*") [5, 203-210] (Fig. 2).

In the course of human history, number systems have become the symbolic representation of sets and also of mechanized procedures which operate on these numbers (arithmetic), in order to merge the different sets behind them (addition), to merge several sets of the same size (multiplication), etc.

According to Husserl, the reason for this is our mental inability to perform these operations directly on the sets [5, 239-240].

3.3 The Stepwise Unfolding of Arithmetic Expressions to the Set Represented by Them

After Husserl the set is the elementary phenomenon and that the representation of numbers in the place-value system is a symbolic representation of this set, from which this set can be recovered at any time. Again, arithmetic expressions are symbolic representations, from which a certain number can be obtained unambiguously. As already mentioned above, Husserl also explains this fact at the end of the second part of his "Logical Investigations" in order to explain the representational meaning of symbols. He explains in an exemplary way (translated from German original):

> "*We make the number* $(5^3)^4$ *clear to ourselves by falling back to the definitory idea: 'Number which arises when one forms the product of* $5^3 \cdot 5^3 \cdot 5^3 \cdot 5^3$*'. If we want to make this latter idea clear again, we have to go back to the sense of* 5^3*, i.e. to the formation* $5 \cdot 5 \cdot 5$*. Going even further back, 5 can be explained by the definition chain* $5 = 4 + 1, 4 = 3 + 1, 3 = 2 + 1, 2 = 1 + 1$." [6]

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3.4 Fulfillment Chains

In the course of the following explanations in [6], Husserl generalizes the step-by-step process of the unfolding of arithmetic expressions described here and postulates that it typically leads to an increase in the richness of content if one, starting from an imagined idea, arrives at an actual representation of a phenomenon over several unfolding steps.

An example of an imagined idea could be the memory of the name of a particular person and the actual representation of a phenomenon could then be to vividly imagine the person to me.

The area of validity of this description shall not be discussed further at this place, but only its applicability to the area of interest here. For this area it can be said without further ado: The transformation of symbolic expressions into musical structures is clearly a process in which a structure containing relatively little information is transformed into a structure with a larger amount of information (see again Figure 1). If this process is also mentally reproduced, this basically corresponds to the scheme of gradually increasing abundance described by Husserl and called "fulfillment chains" ("Erfüllungsketten" GE) by him.

The prerequisite for this information enhancement is always the availability of suitable prior knowledge: With generative grammar, I know how the algorithm works. In the example mentioned above, I remember details of the person to whom the name I came across, refers. In the following we discuss to what extent the arithmetic operations of **AOG** can be performed mentally. The necessary prior knowledge consists on the one hand in the awareness of the corresponding algorithm and on the other hand in our knowledge of arithmetic.

3.5 Phenomenological Investigation

Against the background of the eye-catching parallels of the above example to the unfolding processes described by Husserl with arithmetic expressions, the representation quoted above from Husserl's work is used, so to speak, as a blueprint for the following explanations.

In the examination of the development process described in Chapter 2.2, it is noticeable that the generation of the sequence $t \equiv 7$ can still be easily comprehended. But already here it must be said restrictively that this applies only with exclusive consideration of the first sequence members of this potentially infinite sequence. Also the following multiplication of the resulting sequence by 3 can still be imagined well. At the latest, however, when trying to apply $\equiv 5$ to the preceding result, it becomes very difficult not to lose sight of the previously obtained results.

After all: With pen and paper you can create the score from the symbol series without any problems. Only here, as with every written fixation of a score, there is still the discrepancy between writing and musical interpretation.

Even though in the development of this generative grammar great care was taken to use generally familiar structures and even though the steps in the unfolding process are completely transparent in detail, here one is still far from being able to comprehend the unfolding process in its entirety in the mind. The system of symbols on the highest level with the rules belonging to it enables the composer to produce very complex compositions very quickly. However, the price for this is that a very multi-stage unfolding process has to be passed through in order to come down to the sound level.

Basically, all of this was to be expected, too, if one realizes that in Husserl's presentation the unfolding to a single number and finally to a set is not quite easy and that in the generative grammar introduced here we are dealing with mathematical sequences, which are sets of sets. And the latter do not even form the end point of the unfolding process here, but are followed by the transformation into a score and finally into a musical performance.

Now you can ask yourself how it is even possible to generate a relatively complex score from a few symbols. Where does that come from, what is represented in figure 1 as information growth? - Obviously this unfolding complexity has been bought with a limitation of the amount of possible compositions. Because the fact that grammars of the third order provide for the rule-compliant generation of scores at the same time states that everything in structures that cannot be obtained by applying these rules cannot be represented with the respective grammar either. And what has been said applies to any generative tool. In the present case, music arises with an affinity to (repetitive) minimal music. The musical event is shaped by the metamorphosing structurally related (tone) sequences.

Overall, the use of familiar structures as the basis of a generative grammar is a qualitative prerequisite of being able to imagine the structures unfolding from the representation of symbols, but the practical implementation fails due to the relative limitations of the human imagination.

What has been disregarded in the entire consideration so far is the possibility given today of enabling an immediate sonic implementation of symbol writing via a software in which an arbitrary change of the symbol representation instant is expressed in a corresponding sonic one. Through this feedback mechanism between generative tool and composer, an intuitive knowledge of the direct connection between symbol and sound is established over cycles of intensive use. The compactness of the symbol notation introduced here plays an important role here: it creates a good overview of the entire musical structure on an abstract level and supports the consciously executed influence on the sound event. Even further thought, over time a synaesthesia between symbolic structure, sound and emotional feeling arises, as expressed literarily in the following description of a chess game in Nabokov's "The Defence":

> " *He saw then neither the Knight's carved mane nor the glossy heads of the Pawns – but he felt quite clearly that this or that imaginary square was occupied by a definite concetrated force, so that he envisioned the movement of a piece as a discharge, a shock, a stroke of lightning – and the whole chess field quivered with tension, and over this tension he was sovereign, here gathering in and there releasing electric power.* [14]. "

4 M O D U L O

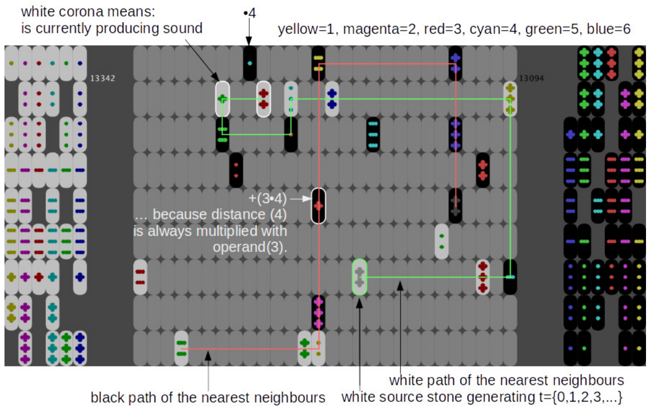

Fig. 3. View of the MODULO playing field.

But there is even another way to learn how to control the power of generative grammar:

In the computer-based board game MODULO (Fig. 3), the players behave consistent to the gaming-rules and do not care about the evolving musical structure. In this respect, MODULO represents an alternative draft to a reasonable and common use of the symbols of the grammar in which the user anticipates the musical result.

In MODULO, the game pieces are arithmetic operations. These are applied along a path of the shortest adjacent distances starting from a source tile representing $id(\mathbb{N}_0)$. Thus, such a path can be understood as a symbolic representation of a piece of music in the sense of the example given in Chapter 2.2.

Above this level is the level of the game rules for the two-person game, who alternately place tiles on the board or move them. The goal of the game is to establish an own path by skilful moves, which consists of operations and operands as mutually different as possible and at the same time to prevent the opponent from doing so. Points are awarded after each move. One way to prove that the rules of the game have been chosen in a meaningful way, as far as the resulting musical result is concerned, is to prove a positive correlation between the number of points achieved by both opponents in a game and the quality of the resulting music. In order to be able to make at least a preliminary statement about this, the game was extended by a component, in which the moves are carried out automatically, whereby from the multitude of possible moves one is always selected, which results in relatively many points. The quality of the resulting sound result can at least be seen intersubjectively in a video [8].

The moves of the opponents have a direct influence on the resulting paths and thus directly on the musical events. A move can have a metamorphosing

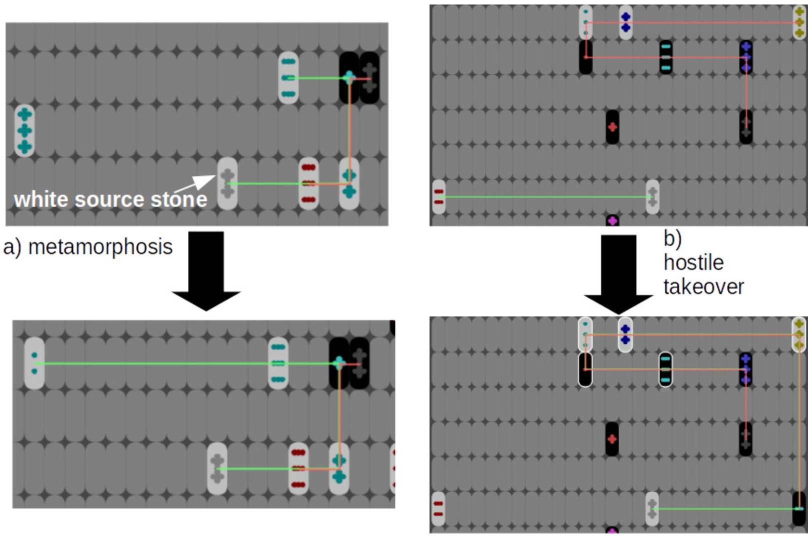

Fig. 4. a) Addition of an operation towards an existing path (metamorphosis). / b) Switching path by adding element close to source tile (hostile takeover).

character (metamorphosis, Fig. 4 a), but it can also cause drastic changes (hostile takeover Fig. 4 b).

5 Summary

It seems impossible in principle that a powerful generative grammar to be presented in a compact way is at the same time designed in such a way that the structures unfolded from it can also be imagined mentally. Theoretically this is possible, but in practice it fails because of the limitations of human imagination. At the same time, an increase in the power of the symbolic language is always linked to a restriction of the overall structures that can be generated. The fact that the symbolic representation does not correspond to the pure phenomenon, but only represents it and thus conceals it, is the reason why generative grammars are powerful tools for the composer, but can in principle not guarantee good control over the sound process, i.e. control based on knowledge. One way of actively establishing the connection between symbolic representation and sound, however, is to present both to the composer coincidentally (real-time composition tool), trusting that the composer can thus learn this connection as intuitive knowledge.

A second way is to make what makes sense measurable and then to give this measure to the composer as feedback and to trust that the composer learns at some point to intuitively maximize this measure through his actions. Such a thing takes place in a sonified, competition-driven performance, if a really meaningful connection between the rules of the game and the sound events has been established. Thus, MODULO is integrated into a series of sonified games in which an attempt is made to establish a clear connection between the course of the game and its musical implementation [16], [4]. As a special feature in comparison to the listed examples it has to be emphasized once again that the game structure and the game rules of MODULO were obtained directly from musical considerations. Specifically, sequences of a grammar based on arithmetic operations are generated with the help of the game moves.

5.1 About Virtuososos and Sumo Robots

Unfortunately, it must be said that this work does not end with the solution of a problem, but in the best case with its sharper contouring:

The possibility to execute real-time composition either leads to trivial results if one has complete knowledge about what one is doing, or symbolically complex actions are triggered, whose non-trivial, but in the best case interesting results will never be completely transparent, especially not in real-time. In fact, however, at least the culture of classical music seems to live from the illusion that the virtuoso interpreter would react spontaneously and knowingly, for example, to the orchestra accompanying him: Through constant repetition of the same phrases in a piece, musicians learn to master a piece of music from a meta-level and can put emotional expression into these phrases, while the actual mechanical process of instrumental playing sinks into the subconscious and is thus mastered perfectly. The recipient, on the other hand, lets himself be drawn into the illusion of a spontaneous, fully conscious play in classical concerts: The enjoyment of a musical performance lies above all in experiencing the totality of technically perfect playing and apparently spontaneous emotional expression as a real fact.

While the virtuoso concert creates the illusion that the musical event unfolds directly from the moment, the illusion in automatically created compositions that are realized in real time lies in the fact that no consciously acting individual is the cause of the musical event. We only project consciousness into the machines [11]. And while in the virtuoso concert the task of the classical composer is above all to anchor the illusion of spontaneity in the structural arrangement of the composition, as a consequence of the preceding considerations the task and special challenge of the developer of real-time composition programs can be seen above all in evoking the illusion of consciously made musical decisions in the recipient.

This is where the embodiment comes into play: A box with a loudspeaker is hardly seen by the recipients as a source for conscious decisions, whereas a humanoid robot, which plays a musical instrument much more likely. This statement needs further explanation: Why do people watch competitions between sumo robots [15]? - It is the fascinating speed with which the opponents (robots) try to push each other from the battlefield. The spectators project consciously acting beings into these opponents. Such a substitution and a transcendental aspect seem to be the two basic ones at most, if not all cultural events: A lot of people come together. On a stage, something emerges that the audience would not normally be capable of representing them. The challenge for those acting on stage is to stage this illusion as perfectly as possible.

5.2 Further Work

Based on the above, there are two possible extensions for the use of a symbolic composition based on arithmetic operations: While retaining the competition as the basic element, which allows for any kind of dramaturgy and evokes an emotional participation of the recipients, two further layers extending the previous concept would be conceivable and have already been partially implemented on a test basis:

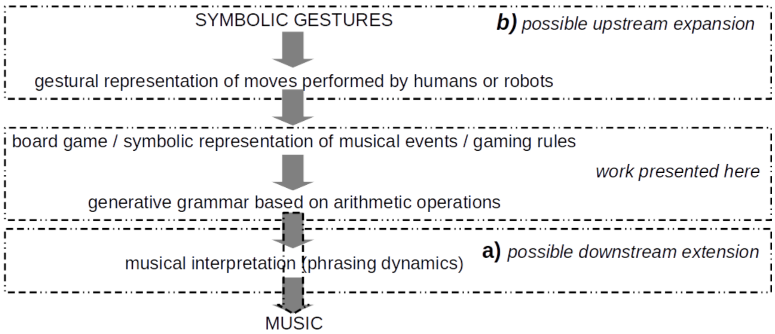

Fig. 5. Possible downstream (a) and upstream (b) expansions.

a) The previous structure can be followed by an interpretative level, which interprets the resulting musical phrases musically, i.e. complements phrasing and dynamics horizontally on the basis of the melody in the individual voices and vertically on the basis of the harmonic development. (Again, there is an increase in information due to prior knowledge, now due to known musical conventions from a certain cultural circle.)

b) An embodiment level can be inserted upstream of the previous structure, which physically implements the actions of the opposing players and thus makes them more tangible for the recipients. While in a) the previous structure can easily be supplemented (see the examples here [9]), in b) the whole structure has to be rethought: Physical movements must replace moves on a playing field. And if the result is not only to end in a mickey mousing, in which certain movements simply evoke certain sounds, but certain gestures become symbols that have a lasting influence on the sound, the symbolic interaction on the playing field must now be reworked into a symbolic, gestural interaction between two opponents. One example for an interaction of two opponents with gestural symbols is the game of rock-paper-scissors, another one can be seen in "tai chi", where interlocking movements take place between certain symbolic fighting postures [19], [20].

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